1. In $\mathbb{R}^3$ with the usual scalar product, consider the subspace U

$$U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : \begin{array}{l} 2x+y-z=0 \\ x-y+3z=0 \end{array} \right\}$$

Compute a basis for $U^\perp$.

2. In a euclidean vector space of dimension 3, the scalar product has Gram matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

with respect to a determined basis B. Let U be the subspace

$$U = \left\{ [\bar{x}]_B = \begin{pmatrix} x \\ y \\ z \end{pmatrix} : \begin{array}{l} x-y=0 \\ x+z=0 \end{array} \right\}$$ Compute $U^\perp$.

3. Let $\mathbb{R}^3$ be equiped with the usual scalar product. Let $\mathcal{E}$ be the standard basis and let $\mathcal{B}$ be the orthonormal basis obtained by applying Gram-Schmidt to $\mathcal{E}$. Find $\underset{\mathcal{B} \leftarrow \mathcal{E}}{P}$.

4. Consider the following equation

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \bar{x} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

and observe that it does not have a solution. Find the best aproximation to a solution.

Hint: Compute the orthogonal projection of $\bar{b}$ onto Col A, where $A\bar{x} = \bar{b}$.

5. Find an orthogonal basis that diagonalizes the map $T(\bar{x}) = A\bar{x}$ where

$$A = \begin{pmatrix} 2 & 2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$